


RAN-2403000503033001
S.Y.B.Sc. (Sem.- III) Examination November - 2025
Mathematics -Major (Paper - MH-MJ-303) Bivariate Calculus
[Total Marks: 50
સૂચના : / Instructions

- (1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
- (2) All questions are compulsory.
- (3) Figures to the right indicate marks of the corresponding question.

Seat No.:

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Student's Signature

Q.1. Answer the following (Any ten)
10

1. Prove that $\sqrt{n+1} = n\sqrt{n}$
2. Obtain $\lim_{y \rightarrow 0} \left\{ \lim_{x \rightarrow 0} \frac{\sin(x+y)}{(x+y)} \right\}$
3. Define Extreme value of a Bivariate function.
4. Evaluate $\int_0^2 \int_1^2 dy dx$.
5. Find the value of $\int_0^1 x^5 (1-x)^3 dx$.
6. Define: Homogeneous function.
7. State the sufficient conditions for the extreme value of a bivariate function.
8. If $x = u + uv^2$ and $y = v + u^2 v$ then find $J(x, y)$.
9. Evaluate $\int_0^1 \int_0^{\frac{x}{2}} x dx dy$.
10. Define: Maxima of a Bivariate function.
11. Obtain f_y for $f(x, y, z) = \frac{y+z}{xz}$
12. Prove that $\beta(l, m) = \beta(m, l)$

Q.2. Answer the following (Any two)
10

1. If f is a differentiable function in x and y such that $xf_x + yf_y = mf(x, y)$ then prove that f is a homogeneous function of degree m .
2. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x+y} \right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.
3. If $x^x y^y z^z = C$, then show that, $x = y = z \Rightarrow \frac{\partial^2 z}{\partial x \partial y} = -[x(1 + \log x)]^{-1}$ where C is a constant.

Q.3. Answer the following (any two) : 10

1. Obtain expression of $e^{ax} \cos by$ in the form of powers of x and y .
2. If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$, then find $\frac{\partial (x,y,z)}{\partial (r,\theta,\phi)}$.
3. Find three positive real numbers whose sum is 27 and product is maximum.

Q.4. Answer the following (Any two) 10

1. Evaluate $\iint_S (6 - x - y) dx dy$, where $S = \{(x,y) / x \geq 0, y \geq 0, x + y \leq 6\}$.
2. $\iint_S x^2 y dx dy$ where S is a circle $x^2 + y^2 = a^2$ in the first quadrant.
3. Change the order of integration of the double integral $\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}} f(x,y) dy dx$.

Q.5. Answer the following (Any two) 10

1. Prove the following:
 (i) $B(l, m) = 2 \int_0^{\frac{\pi}{2}} \sin^{2l-1} \theta \cdot \cos^{2m-1} \theta d\theta$. (ii) $B(l, m) = \int_0^{\infty} \frac{y^{l-1}}{(1+y)^{l+m}} dy$
2. State and prove the relation between Beta and Gamma function.
3. Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$.
